

A STUDY OF THE ACHILLES TENDON WHILE RUNNING

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Abstract

The following study attempts to elaborate a model of the Achilles tendon while in the process of running, specifically during a step that is part of a running sequence. Data are collected with the help of a force plate and then is processed and modeled to serve as a starting point and comparison to a mathematical model using polynomial functions.

The data collected were filtered to diminish recording of “noise” and an empirical model was established. Mathematical models using second order and fourth order polynomials were employed, as well as an approximation using known maximal force. The increase in the accuracy of modeling was determined as the order of the polynomial function increased.

Achieving an accurate predictor function is essential in understanding the biomechanics of the Achilles tendon.

Keywords: force plate, Achilles tendon, ground reaction force, polynomial function, impulse.

Introduction

Traumatic Achilles tendon tears are frequent occurrences in lower-limb trauma, involving young, active patients [1]. To better understand the etiology and etiopathogenesis of tendon tears, a comprehensive model of the Achilles tendon behavior during rupture-risky daily situations and motions is needed. Analyzing the biomechanics of the Achilles tendon requires in fact a „dissection” of the main kinematic events during a running sequence.

The Achilles tendon structure allows it to transmit significant forces from muscle to bone, behaving like a hyperelastic material past a certain longitudinal strain, but shows a limited resistance to shearing and compression loads [2,3]. At rest, tendon fibers appear wavy, whilst during strain they become taut, behaving like an elastic material up to 2% elongation; past that, they behave like a hyperelastic material. Partial tears develop at 4% elongation and beyond, while complete tears usually occur at 8% elongation and beyond [4,5].

The forces acting on the foot can be measured experimentally with the aid of a force plate, a device that utilises Newton’s Third Law [6]. By recording the ground reaction force (GRF), we can determine the forces that act

on the Achilles tendon while running, in particular those acting on the tendon-calcaneus junction [7].

A running sequence is much more complex compared to a walking one, completely lacking in bilateral support and having an intermediate phase of „flight”, when the body is completely airborne [8]. The Achilles tendon behaves like an interface between muscle and bone during such a sequence, establishing the so called calcaneus-tendon-sural triceps system. If we consider the bony surface as completely inelastic, the forces developed by the triceps surae input a maximum load on the tendon’s insertion [9,10]. Understanding and predicting these forces during the ground contact phase of a running step is paramount to better understanding the impact of running on the Achilles tendon.

Materials and Methods

To record the ground reaction force (GRF), a Kistler Force Plate was used, with a recording frequency of 2.5 KHz; a 23-years-old human subject with a weight of 77 kg, a height of 182 cm and with no prior tendon injuries and no prior specific physical training was asked to run over the force plate 30 times. The collected data were filtered and statistically analysed, ignoring any force past 5 SDs (standard deviations), rendering a force cloud that was sorted in regards to time:

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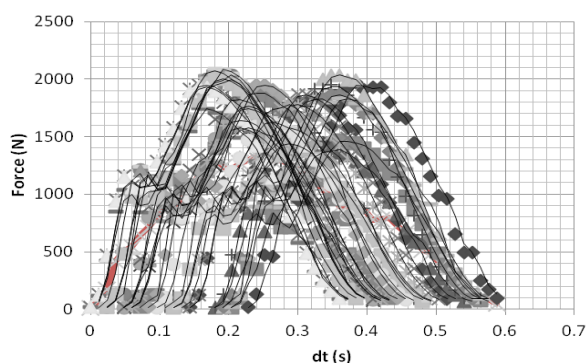


Figure 1. Force cloud results and median (red).

It became readily apparent that the median (red line) is a rough, intuitive image of the running function model. A median regression function applied to the force cloud yields a clearer image:

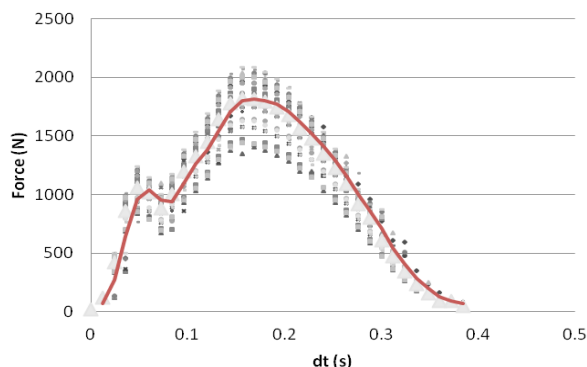


Figure 2. Median regression function applied to the force cloud.

In order to elaborate and compare a mathematical model to an empirical one, we chose the running sequence that was closest to the median regression curve. Although one could easily use the regression function as a model itself, we felt that a mathematical inference model was much more useful to appreciate the biomechanics of the Achilles tendon.

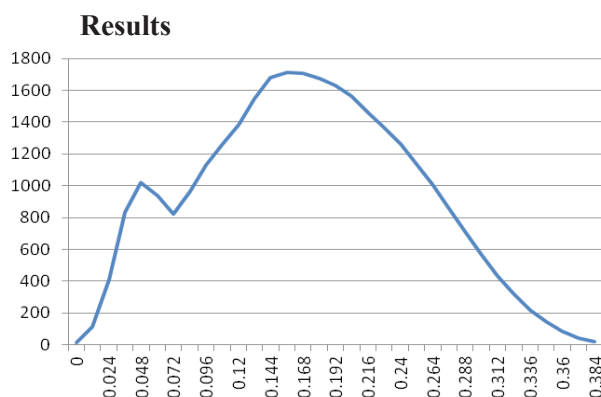


Figure 3. Empirical model of a running step.

Given that $F=M \cdot a$, where F stands for force, M for mass and a is the gravitational pull (9.81 m/s^2), F for our subject amounts to 755.3 N . The area below the curve is in fact Impulse (I), which can be calculated using the Riemann sums:

$$I = \int F \cdot dt \approx \sum_{k=1}^n \frac{F_k + F_{k+1}}{2} \cdot \Delta t;$$

where F_k is the force recorded at the moment t_k , with $T=0.384$ and $\Delta t=t_k-t_{k-1}$ being constant (0.004 s). An impulse of $I=360.31 \text{ Ns}$ results. This can be used to compare the accuracy of our mathematical model to the empirical data.

A first modeling attempt is to try and approximate the function with the help of a second degree polynomial, $f(t) = -at^2+bt+c$, the first term being negative because the function is concave. The impulse

$$I = \frac{-aT^3}{3} + \frac{bT^2}{2} + cT,$$

is obtained through imposing limits $f(0)=0$ and $f(T)=0$, thus the initial equation becoming $f(t) = -a(t^2-Tt) = -at(t-T)$, with only „a” unknown. Impulse becomes:

$$I = \int_0^T f(t) dt = \frac{aT^3}{6}.$$

Empirical data shows that the maximal value of the function is satisfied at the middle of the interval, $f\left(\frac{T}{2}\right) = f_{\max}$ and $f'\left(\frac{T}{2}\right) = 0$, which is actually obvious as we purposely chose to model using a 2nd degree function.

Knowing that:

$$f_{\max} = f_{\text{experimental}}\left(\frac{T}{2}\right), \quad f(t) = -\frac{4f_{\max}}{T^2}t(t-T)$$

and impulse $I = \frac{2f_{\max}T}{3}$, we can calculate the impulse using maximal force (known empirically). Thus, $T=0.384$ and maximal force is $1,711.66 \text{ N}$, the resulting impulse $I=438.18 \text{ Ns}$, which is close, but not sufficiently so the empirically devised impulse.

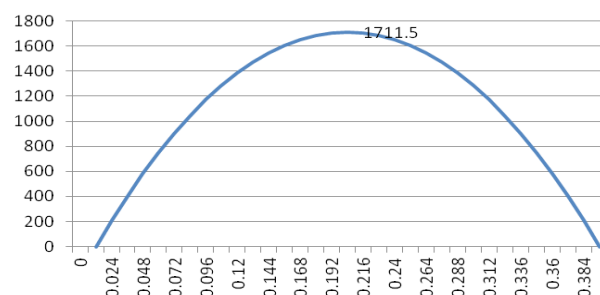


Figure 4. Known maximal force modeling.

Using body weight to model the function can be useful, as in mathematical terms, body weight (BW) can be described with the median of the function (during a sprinting step, the entire body weight is shifted from one

foot to another, thus during a whole step the entirety of the weight is supported):

$$\bar{f} = \frac{1}{b-a} \int_a^b f(x) \cdot dx$$

$$\int_0^{t_1} (BW - f(t))dt + \int_{t_1}^{t_2} (BW - f(t))dt = \int_{t_1}^{t_2} (f(t) - BW)dt$$

where t_1 and t_2 represent the crossing of the curve by the line that expresses the median of the function. Thus, BW can be described as:

$$BW \cdot t_1 - \int_0^{t_1} f(t) \cdot dt + BW(T - t_2) - \int_{t_2}^T f(t) \cdot dt = \int_{t_1}^{t_2} f(t) \cdot dt - BW(t_1 - t_2)$$

$$\Rightarrow BW \cdot T = \int_0^T f(t) \cdot dt \quad (BW = (\int_0^T f(t) \cdot dt) / T = I / T)$$

An obvious relation between weight and impulse is described, but as we can see by using previous results,

$$BW = \frac{2f_{max}}{3}, \text{ which in turn means } f_{max} = \frac{3}{2}BW$$

Using a fourth order polynomial:

$$F(t) = at^4 + bt^3 + ct^2 + dt + f, \text{ where } t \in [0, T],$$

with the limits $F(0) = 0$, $F(T)=0$, $F'(0)=0$, $F'(T)=0$ and $F(\frac{T}{2})=f_{max}$, leads us to:

$$F(t) = \frac{16f_{max}}{T^4} t^2(t-T)^2, \text{ which results in}$$

$$f_{max} = \frac{15}{8}BW, \text{ a rather accurate description.}$$

In the end, by comparing the different methods of approximation, we can observe that a known maximal force approximation seems to be the most accurate:

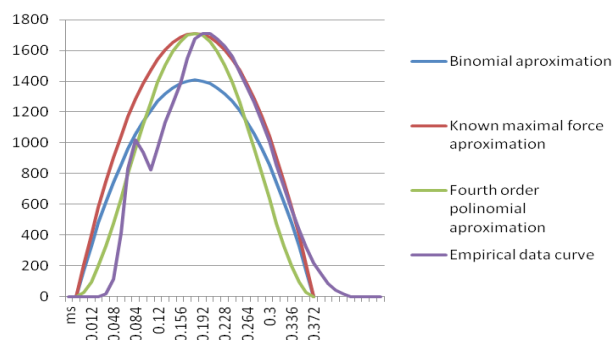


Figure 5. Comparable modeling.

Discussion

Obtaining a perfectly accurate model of the step in a running sequence is a daunting task if attempted through mathematical means alone; the higher the order of the polynomial, the more accurate the modeling gets, but this continues *ad infinitum*, because it is done by approximation. It needs not to be perfect to serve a predictive purpose though: having established a satisfactory model and

developing a quick, repeatable and easy way to provide it is a sufficient goal if one seeks to understand the behaviour of the foot in a running sequence and through it, the strain and loads incurred on the Achilles tendon – calcaneus joint.

Strictly from a mathematical point of view, a fourth order polynomial seems to be sufficient to model running; a further increase in order will only „slim” the curve of the approximation, making the transition to the peak force steeper – while useful to extract a more accurate impulse and therefore load, it diminishes the inequality of force distribution, opposite to the empirical data gathered that reveals the fact that the running step is in fact a walking step in a much shorter time span. A high order polynomial approximation function suggests that the step is a sprinting step, where ground contact is achieved only on the forefront of the foot, toe region and distal metatarsal region, foregoing the calcaneus landing altogether. It is probably more accurate to divide running into light running (which was the object of the study) and sprinting, where the foot behaves differently.

As for inaccuracy sources, recording “noise” when collecting data can probably be considered the main source of error, while the presence of medio-lateral and antero-posterior forces unaccounted for during data collection and filtering could come in second place. When modeling a foot that has a distinctly altered biomechanics due to malformations, recent surgery, pain, neuromotor injuries etc., care should be taken in ignoring the concurrent forces; in a healthy foot these forces are negligible in a running sequence.

Conclusions

Building an accurate model of running allows to identify key kinematic moments and events that impact the foot and specifically, the ankle with its main muscle/bone system that is the triceps surae – Achilles tendon – calcaneus complex.

The heel region in general and the Achilles tendon in particular bears loads equivalent to twice and sometimes more than twice the body weight during running. Moreover, the impulse generated by these loads is distributed unequally throughout the running sequence, creating high-strain events for the Achilles tendon.

The most important and impacting force during running is the vertical traction force (as described, through Newton’s Third Law, by the ground reaction force). By establishing and describing the relationship between body weight, ground reaction force and impulse in each step of the running sequence, we can model the entire kinematic chain. A polynomial model, although rather intricate, can provide a useful tool in predicting and assessing the behavior of everyday movements, in this particular case during sprinting and running. By recording the movement, one can simply extrapolate and compare between results from healthy individuals or between similar tendon healing

rates in traumatic tendon injuries; pathological motions can be detected as well as abnormal loads or prolonged loading times which can suggest unfit tendons or slow functional healing.

One must remember that while the vertical traction force is the main actor while running, medio-lateral and antero-posterior forces can be detected, even though minimal, they can impact the Achilles tendon and the overall motion, if for some reason their value is greatly increased through pathological movement or antalgic motions. The Achilles tendon is poorly equipped to deal with shear and compression forces, which can be cause for re-rupture. Evaluating these forces requires more accurate recordings and an ability to replicate the motion precisely.

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